

A Brief Discussion on the Connection Between Secondary School Mathematics and University Mathematics from a Higher Perspective: Taking Probability Theory as an Example

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Abstract. The paper addresses a concrete question: how can secondary mathematics prepare students for the formal treatment of probability they will encounter later? We begin by diagnosing two persistent constraints in secondary classrooms—content that rarely moves beyond elementary rules and pedagogy that privileges worked examples over reasoning about uncertainty. Using Piaget's account of developmental readiness together with constructivist and Vygotskian perspectives, we derive criteria for selecting and staging ideas from higher probability (e.g., conditional reasoning and Bayesian updating) so that they remain conceptually faithful yet instructionally accessible. On this basis, we propose an integration toolkit that couples modest extensions of content with inquiry-oriented methods and targeted professional learning. The toolkit is instantiated in a teaching case on "Probability Problems in Disease Testing," which models how secondary students can make and revise likelihood judgments. The intended contribution is practical rather than rhetorical: a feasible pathway that may heighten interest, support mathematical literacy, and improve problem-solving transfer, thereby advancing a more coherent mathematics sequence.

Keywords: Educational Reform, Higher Mathematics, Secondary School Mathematics, Probability Theory

1. Introduction

The Full-time Compulsory Education Mathematics Curriculum Standards (Experimental Version) issued in 2001 proposed that students should acquire the mathematical knowledge, ways of thinking, and application skills necessary for adapting to future societal life and further development [1]. The cultivation of disciplinary ways of thinking plays a key role in enhancing learners' thinking quality, cognitive abilities, and comprehensive competence. As the primary venue for achieving this goal, classroom instruction requires teachers to integrate mathematical thinking throughout the entire teaching process. In response to the practical pathways for implementing ways of thinking in classroom teaching, educators have summarized several strategies. The core points can be outlined as follows: teachers should consciously foster mathematical thinking, systematically plan instruction based on the disciplinary knowledge system, curriculum standards, and student characteristics; optimize the structure of teaching materials and lesson designs to deepen theoretical understanding of mathematical ways of thinking; emphasize the natural integration of thinking methods during

knowledge transmission; and seize key teaching opportunities for organic infusion, ultimately constructing a teaching paradigm that promotes the coordinated development of knowledge systems and thinking methods [2].

With the deepening of mathematics curriculum reform, a large number of modern mathematical elements have been incorporated into current secondary school textbooks, placing higher demands on educators to extract and cultivate new forms of mathematical thinking. In 2014, the national education authorities issued the Opinions on Comprehensively Deepening Curriculum Reform and Implementing the Fundamental Task of Moral Education, which for the first time established the “core competency system” as the core goal of educational reform [3]. Against this backdrop, cultivating core competencies in disciplines has become the mission of contemporary educators, and fostering core mathematical competencies has become both the fundamental task and the ultimate pursuit of mathematics education.

The General High School Mathematics Curriculum Standards (2017 Edition) emphasize in the curriculum implementation recommendations that teachers should grasp the mathematics curriculum system from a holistic perspective, and through thematic and unit-based instructional design and practice, truly implement the requirements for cultivating disciplinary core competencies [4]. This unit-based instructional model, oriented toward core competencies, breaks through the limitations of traditional point-by-point knowledge teaching and advocates for structuring and integrating instructional content around major disciplinary concepts.

Within mathematics, probability is best understood as a language for reasoning under uncertainty rather than an isolated topic. Three pathways are especially relevant to the present study. First, the calculus of random variables and distributions provides the scaffolding for statistical modeling and inference; it explains why data-analytic claims depend on assumptions about variability and sampling rather than on arithmetic alone. Second, ideas from stochastic processes clarify dynamic uncertainty—how today's information updates tomorrow's beliefs—and thereby connect routine classroom work (e.g., conditional reasoning) to later formal tools used in information systems and economic decision models. Third, probability's dialogue with analysis has sharpened both fields: limit theorems motivate measure-theoretic methods, while analytic techniques, in turn, discipline probabilistic convergence. As a side current, the “probabilistic method” has also reframed selected problems in number theory by treating structure through typical behavior rather than deterministic construction. These examples are illustrative rather than exhaustive; they motivate a focused, age-appropriate selection of higher-level ideas for secondary classrooms.

At the secondary level, the bridging role of probability is pedagogical as well as conceptual. When linked deliberately to combinatorics and to descriptive statistics, probability helps students model, quantify, and revise uncertainty in everyday contexts (for instance, interpreting a diagnostic test, evaluating a reliability claim, or comparing competing risks). The intended outcomes are observable: students articulate assumptions, compute and explain conditional relationships, and justify choices under uncertainty. For transition to higher study, we propose criteria for staging content rather than simply “adding more”: prioritize concepts that (i) generalize familiar ideas (e.g., from relative frequency to conditional probability), (ii) prepare later formalism without overloading notation (e.g., distributions as mappings from outcomes to likelihoods), and (iii) invite inquiry methods such as simulation and counterexample. Under these criteria, secondary courses can build a cognitive foothold for subsequent work in probability and statistics while laying a realistic runway toward data-centric fields (e.g., data science, operations and finance) without claiming premature mastery.

To address the issue of the connection between secondary and university mathematics, F. Klein proposed that the concept of functions should be emphasized in secondary school mathematics teaching and that calculus should be incorporated into the goals of secondary mathematics curricula. In his work *Elementary Mathematics from an Advanced Standpoint*, he defined the natural logarithm

using $\int \frac{1}{x} dx$, linking the natural logarithm with integration, and, by referencing Bürgi's historical methods for calculating logarithmic tables, proposed an alternative definition of the logarithmic function. To enable high school students to gain a more comprehensive understanding of the concept of logarithmic functions, researchers developed a sequence of tasks to help students apply logarithmic functions more flexibly [5].

We start from a restricted set of ideas in probability and mathematical statistics that are usually formalized at the university level (e.g., conditional reasoning, distributional thinking, and the logic of inference) and re-express them in secondary-appropriate representations and tasks. The aim is not to transplant tertiary formalism but to stage concepts that prepare later study: connecting relative frequency to conditional probability, treating distributions as mappings from outcomes to likelihoods, and modeling how evidence updates beliefs. Within this design, probability and statistics are woven into existing secondary topics through worked contexts and short simulations, so that students articulate assumptions, compute and explain conditional relationships, and justify decisions under uncertainty. In doing so, the course builds a coherent runway from secondary to university mathematics without overloading notation or scope.

2. Overview of probability in advanced mathematics

In university programs—especially in science and engineering—probability and mathematical statistics provide the formal language for reasoning about randomness. Together with analysis and linear algebra, they supply methods that underwrite later specialized study. Typical curricula are organized in two linked modules: a theoretical module that develops probability (events, random variables, distributions, limit behavior) and an applied module that develops statistics (estimation, testing, inference), so that methodological practice rests on explicit probabilistic assumptions rather than on procedures alone.

Historically, the field has advanced in stages that clarify both scope and tools. Classical probability worked in finite spaces under symmetry assumptions and relied on combinatorics to count equally likely cases. Modern probability extended the domain to continuous phenomena via calculus, introducing distributional descriptions and moment calculations for random variables. Contemporary probability adopted a measure-theoretic foundation, yielding an axiomatic system that unifies discrete and continuous cases and explains convergence and limit theorems with precision. Most higher-education courses are anchored in this modern–contemporary framework, hence their tight connection to analysis.

The interface with analysis is structural rather than superficial. Analysis is built around functions, limits, and continuity; probability reinterprets these ideas through random variables—measurable functions from an underlying sample space to the reals—so that uncertain outcomes are rendered as numerical objects amenable to calculus. Specifically, let the sample space of a random experiment E be S , and let a random variable X be a mapping that assigns a real value $X(e)$ to each sample point e in S . Then, for any set of real numbers $A \subset \{X(e) | e \text{ is a sample point}\}$, the relation $P \subset \{X \in A\} = P(X^{-1}(A))P$ holds, where $X^{-1}(A)$ denotes the set of sample points $e \in S$ for which $X(e) \in A$. For discrete random variables, the theory of infinite series demonstrates unique advantages in calculating numerical characteristics such as mathematical expectation and variance, with convergence theorems for series serving as critical criteria for verifying the existence of such values [6]. Seen this way, probability does not merely borrow tools from analysis; it organizes them around uncertainty, which explains the field's methodological reach across data-centric domains.

For the purposes of this study, these connections justify a selective emphasis at the secondary–tertiary interface: focusing on distributional thinking, conditional reasoning, and simple convergence ideas in representations that are faithful to the higher framework yet instructionally accessible. The

goal is not to compress university formalism into secondary courses, but to prepare a coherent pathway that makes later measure-theoretic and statistical developments intelligible when students encounter them.

3. Analysis of the current status of probability teaching in secondary schools

3.1. Knowledge and skills

Secondary mathematics is typically taught through deterministic lenses—algebraic manipulation and geometric proof admit unique answers under fixed premises. Probability asks students to reason differently: outcomes are uncertain, yet patterns emerge in aggregates. A simple die-rolling context makes the contrast concrete. Although a single roll is unpredictable, repeated trials bring relative frequencies into alignment with the theoretical model, giving students an empirical route to the idea that stability can arise from randomness.

Problem solving in probability, therefore, relies on both structural representation and logical control. Students must specify a sample space, define events, and organize cases without overlap before any calculation is meaningful. In classical settings this means partitioning possibilities and checking that counts reflect the structure of the situation rather than ad hoc listing. Alongside this deductive organization, students also practice inductive habits: by observing data from repeated experiments, they form a statistical interpretation of probability and encounter the tendency of frequencies to settle—an accessible precursor to large-sample regularities often formalized later (e.g., laws of large numbers) [7]. The productive tension between these two modes—deductive structure and inductive stabilization—builds a skill set that is distinct from, yet complementary to, purely deterministic work.

3.2. Mathematical literacy

Within the secondary curriculum, probability broadens what it means to “do mathematics.” It signals that mathematics models not only certainty but also uncertainty, providing tools to quantify, compare, and revise beliefs. This expansion is conceptual and methodological. Conceptually, students move from rules for exact calculation to frameworks for reasoning under risk; methodologically, they adopt representations (events, random variables, distributions) that tie empirical observation to general statements.

The knowledge structure is layered rather than flat. From foundational principles and classical models through distributional thinking, students build a pathway that links combinatorics (counting principles that support probability operations) with geometric ideas (spatial viewpoints used in probability measures and visual models). These links are not ornamental; they help students see algebraic and geometric tools as resources for reasoning about chance, thereby enriching their overall mathematical cognition [8].

Applications naturally invite cross-disciplinary integration. Geometric probability, for instance, requires students to coordinate measurement, coordinate systems, and proportional reasoning with probabilistic definitions to construct spatial models of chance. Such tasks foster transfer: learners shift among representations, justify assumptions explicitly, and choose tools appropriate to the uncertainty at hand. In this way, probability supports the development of mathematical modeling and comprehensive problem-solving—not by adding isolated topics, but by organizing familiar techniques around questions of variability and evidence.

4. Theoretical foundations for integrating probability into secondary mathematics

We ground the proposed integration in three complementary perspectives—Piagetian development, constructivist learning, and Vygotsky's zone of proximal development. Together they motivate what to introduce from higher probability, when learners are ready, and how to stage ideas so that representations remain age-appropriate and instructionally actionable.

4.1. Piaget's theory of cognitive development

By secondary school many students operate within the formal operational range: they can coordinate symbols, entertain hypotheticals, and reason with conditional statements. Probability topics leverage these capacities but also extend them. Concepts such as conditional probability, independence, and Bayesian updating require students to separate what is possible from what is plausible given evidence, and to justify transitions between the two.

In classroom terms, tasks should invite both deductive control and inductive stabilization. Deductively, students specify a sample space, define events, and partition cases before computing—skills that strengthen logical discipline. Inductively, they observe data from repeated trials or simulations to see how frequencies settle toward model expectations, building an empirical bridge to later formal results. Contexts like the Monty Hall problem or diagnostic testing make these modes visible: students articulate competing hypotheses, compute conditional relationships, and then check conclusions against simulated evidence. Such work aligns with formal-operational capacities without presuming university-level formalism and supports early habits of model-based reasoning.

4.2. Constructivist learning theory

Constructivism views learning as the active construction of meaning from prior ideas and experiences. For probability, this suggests staging advanced ideas from familiar anchors: from relative frequency to conditional probability (e.g., two-way tables and tree diagrams), from counting outcomes to distributional thinking (e.g., repeated Bernoulli trials), and from single answers to evidence-based revision (e.g., updating beliefs after new data).

Illustrative pathways keep everyday contexts central. A short investigation on winning streaks or quality control can motivate the binomial model; arrival-rate scenarios can gesture toward Poisson-like reasoning when counts are rare and intervals are fixed—without overloading notation. Inquiry cycles—posing a question, simulating (including simple Monte Carlo with spreadsheets or apps), comparing predictions with outcomes, and reflecting on mismatch—help students externalize assumptions and revise them. In this design, probability is not delivered as facts but constructed through tasks that connect data, representation, and explanation.

4.3. Vygotsky's Zone of Proximal Development

Vygotsky's ZPD highlights the gap between what learners can do unaided and what they can do with guidance. Many “higher” probability ideas fall within this zone when scaffolded appropriately. We translate the principle into design moves:

Diagnostic entry: use brief probes (e.g., contingency-table items, tree-diagram completions) to locate students' current control of prerequisite ideas such as event definition, counting, and basic conditional statements.

Tiered tasks: offer a core task (e.g., computing $P(A \mid B)$ from a table) with extensions (interpreting odds ratios, contrasting $P(A \mid B)$ with $P(B \mid A)$, or exploring base-rate effects) so that challenge stays inside the ZPD for diverse learners.

Representation scaffolds: move from concrete records (tallies, simulations) to structured diagrams (trees, two-way tables) to compact formulas, with worked-example fading and prompted explanation to manage cognitive load.

Collaborative sense-making: pair explanations and peer critique so that students verbalize assumptions and check for independence/conditioning errors.

Applied this way, ZPD-aligned instruction converts potential into performance: students not only carry out calculations but also justify their modeling choices and identify when conditional statements are being misapplied. The emphasis is on gradual release—from guided representations to independent reasoning—so that secondary classrooms prepare a credible runway to later probability and statistics.

5. Integration strategies and instructional case design

5.1. Strategies for integrating and expanding instructional content

We extend secondary probability by selectively importing higher-level ideas that (i) generalize what students already know, (ii) admit accessible representations, and (iii) prepare later formal study without excessive symbolic load.

Conditional probability and Bayes, staged. Begin with two-way tables and tree diagrams to contrast $P(A | B)$ with $P(B | A)$ and to surface base-rate effects. Real contexts—diagnostic testing or spam filtering—anchor the prior $\rightarrow$ evidence $\rightarrow$ posterior narrative. Only after students can compute and explain conditional statements from tables do we compress to formulas (multiplication rule, Bayes update). Short “intuition vs. calculation” prompts make the gap visible and productive.

Expectation with structure. Treat expected value as a linear operator ($E[aX + bY] = aE[X] + bE[Y]$) rather than a single arithmetic average. Use small portfolios or reliability combinations to show how linearity supports “diversification” reasoning without invoking variance formally. Students state assumptions (independence, additivity of payoffs) before applying linearity, then check reasonableness against a quick simulation.

Stability by simulation. Simple Monte Carlo experiments (e.g., coin tosses, sampling from a bag) let students observe frequency stabilization as trials grow. The emphasis is conceptual: long-run regularity does not predict single trials. Students articulate this distinction in writing and connect it to classroom rules for inference.

Distributions as families, not formulas. Binomial models arise from repeated independent trials with fixed success probability; Poisson models are introduced as a rare-event limit when counts occur over equal intervals. Contexts such as call arrivals or defect counts motivate the move without measure-theoretic machinery. Students compare histogram shapes across parameters, describe center and spread qualitatively, and explain which assumptions link a context to a model.

To keep the extension actionable, each topic follows a representation ladder—data table $\rightarrow$ diagram $\rightarrow$ compact formula—paired with two accountability moves: (a) a contrastive pair that forces students to distinguish $P(A | B)$ from $P(B | A)$, and (b) an explain-first prompt where students write assumptions before calculating. Misconceptions (e.g., base-rate neglect, double-counting, independence misreads) are anticipated explicitly and addressed with targeted counterexamples.

5.2. Innovation and application in teaching methods

Method choice is aligned to the cognitive work required in probability: making assumptions explicit, coordinating representations, and revising beliefs with evidence. We combine problem-driven, inquiry, and lightweight project structures in a common cycle—Launch $\rightarrow$ Investigate $\rightarrow$ Consolidate $\rightarrow$ Transfer.

Problem-driven launch. Start with a decision question that cannot be answered deterministically (e.g., "Given today's rain, how should a forecast for tomorrow change?"). Students commit to a prediction and a rationale, then translate the question into a two-way table or tree. As complexity increases (e.g., diagnostic testing with known accuracy and prevalence), Bayes' Theorem is introduced as a compression of the reasoning they already performed, not as a black-box formula. This sequence cultivates conditional thinking and keeps the mathematics tethered to decisions [9].

Inquiry with simulation. For questions like "How does the number of tosses affect the stability of the head proportion?", students design the experiment (trial counts, recording scheme), run simulations (manually or with simple digital tools), and plot frequency traces. When traces deviate from expectation, the class interrogates why—measurement procedures, hidden biases, or sampling variability—thereby reinforcing the difference between model idealizations and physical implementations. Students finish by writing a two-sentence rule: what stabilizes, and what does not.

Project-based consolidation. Small teams take a context (quality control, reliability of a two-component system, screening in a low-prevalence setting) and produce an evidence brief: assumptions, model choice (binomial vs. Poisson), representation (table/tree/graph), calculation or simulation, and a one-paragraph recommendation with caveats. Roles (modeler, checker, explainer) distribute cognitive load; peer review targets common errors (reversing conditionals, ignoring base rates).

Assessment for reasoning, not just answers. Exit tickets ask students to (i) restate a conditional claim in words with its conditioning event named; (ii) draw a quick tree/table for a new but similar context; (iii) note one assumption that, if violated, would change the result. These artifacts make probabilistic reasoning observable and provide data for the next day's scaffolds.

Across methods, the design principle is constant: represent first, compute second, justify always. Students leave not only with correct numbers but with traceable arguments about uncertainty and evidence.

5.3. Instructional case design

Teaching Probability Thinking through Bayes' Theorem and Disease Screening	
Teaching Objectives	Knowledge Objectives:
	1. Enable students to understand the core concepts of conditional probability and Bayes' Theorem.
	2. Help students master the method of calculating "inverse probability" using Bayes' formula.
	Skills Objectives:
Key and Difficult Teaching Points	1. Enable students to understand the core concepts of conditional probability and Bayes' Theorem.
	2. Help students master the method of calculating "inverse probability" using Bayes' formula.
	Affective Objectives:
	1. Guide students to appreciate the value of probability theory in real-life decision-making and to strengthen their awareness of rational decision-making.
	Key Points: Derivation and application of Bayes' Theorem.
	Difficult Points:
	1. Guide students to understand the relationship between "prior probability" and "posterior probability."
	2. Help students overcome intuitive misconceptions, such as confusing test accuracy with the probability of being ill.

Teaching Procedures	1. Introduction (1) Play a news clip about “early cancer screening,” then ask students: “Does a positive test result mean the person is definitely ill?” (2) Present the data: A certain disease has a prevalence of 1%, and the test has 95% accuracy (i.e., 95% of patients test positive, and 5% of healthy individuals yield false positives). Ask students to raise hands to vote on the estimated probability of actually being ill given a positive result. Record typical answers (e.g., 95%). (3) Reveal the actual probability ($\approx 16\%$) and highlight its contrast with intuitive guesses, sparking students’ curiosity and motivation for further exploration. 2. Exploration and Explanation (1) Review the basics: Prompt students to recall the conditional probability formula: $P(A B) = \frac{P(A \cap B)}{P(B)}$. Reinforce understanding with a real-life example: “Given that a die shows an even number, what is the probability it is greater than 4?” (2) Introduce Bayes’ Theorem: Break down the disease screening scenario: Prior probability: $P(\text{disease}) = 0.01$ Likelihood: $P(\text{positive} \mid \text{disease}) = 0.95$; $P(\text{positive} \mid \text{healthy}) = 0.05$ Use a probability tree or contingency table to compute total probability: $P(\text{positive}) = P(\text{disease}) \cdot P(\text{positive} \mid \text{disease}) + P(\text{healthy}) \cdot P(\text{positive} \mid \text{healthy})$ Derive Bayes’ formula: $P(\text{disease} \mid \text{positive}) = \frac{P(\text{disease}) \cdot P(\text{positive} \mid \text{disease})}{P(\text{positive})}$. Substitute the data to calculate and verify the result ($\approx 16\%$) and compare with students’ initial guesses, prompting reflection. 3. Reinforcement and Application (1) Variations: Present two variant problems: If the disease prevalence increases to 5%, what is the probability of being ill after a positive result? (Answer $\approx 49.3\%$) If the test is taken twice and both results are positive, how does the probability change? (Assume test independence.) Organize group discussions to analyze the reasons behind the changes and summarize how prior probability influences posterior probability. (2) Interdisciplinary Case: Use spam email filtering as an example. Pose the problem: “Given that an email contains the word ‘free,’ what is the probability it is spam?” Provide data: In historical data, 30% of emails are spam; 80% of spam emails contain the word “free,” and 10% of normal emails do. Have students work in groups to compute the posterior probability and discuss the rationality of the algorithm’s design. 4. Summary (1) Knowledge recap: Review the formula and concepts of Bayes’ Theorem, emphasize the correction process of “inverse probability,” and highlight the importance of prior information and the chain rule of conditional probability. (2) Cognitive development: Encourage students to critically assess intuitive judgments, stress the value of mathematical tools in complex decision-making, and give examples of Bayes’ Theorem applications in AI, medicine, law, and other fields. (3) Homework: Assign a basic exercise for students to calculate the probability of “a negative result but actually being ill.” Assign an extended task requiring students to design a real-life Bayes’ problem (e.g., updating weather forecasts) and write an analytical report.

6. Conclusion

This study examined a practical question: how can secondary mathematics prepare students for the formal treatment of probability they will later encounter at university without importing excessive symbolism or scope? Using probability as the testbed, we proposed a selective integration that privileges ideas with clear secondary representations—conditional reasoning, distributional thinking, and evidence-based updating—and staged them through a representation ladder (tables/trees $\rightarrow$ graphical summaries $\rightarrow$ compact formulas). A worked case on diagnostic testing illustrated how prior–evidence–posterior reasoning can be taught as an extension of familiar tools rather than as a detached theorem.

The intended contribution is twofold. Instructionally, the approach makes probabilistic reasoning observable: students articulate assumptions, distinguish $P(A \mid B)$ from $P(B \mid A)$, compute and explain conditional claims, and revise judgments after new evidence. Curricularly, it offers a credible

bridge from secondary topics (combinatorics, descriptive statistics) to higher study in probability and statistics by organizing familiar techniques around uncertainty rather than around procedures alone.

Three theoretical perspectives justify the design while constraining its ambition. Piagetian accounts support work with hypotheticals and conditionals typical of the formal-operational range; constructivism motivates inquiry cycles in which students build concepts from experiences and representations; Vygotsky's ZPD argues for scaffolded challenge—diagnostic entry, tiered tasks, and gradual release—so that higher-level ideas remain instructionally accessible rather than symbolically overwhelming [10].

Implementation follows a consistent cycle—Launch → Investigate → Consolidate → Transfer. Problem launches anchor decisions under uncertainty; investigations use simulation to separate single-trial unpredictability from long-run regularity; consolidation compresses reasoning into formulas (e.g., multiplication rule, Bayes' update) after ideas are publicly shared; transfer tasks ask students to justify model choice and note conditions under which conclusions would change. Teacher learning is integral: professional development should target conditional language, common misconceptions (base-rate neglect, conditional reversal, independence errors), and the management of representation shifts.

This proposal has boundaries. It does not claim mastery of university formalism nor evaluate outcomes through controlled trials. Its value lies in feasibility and traceability: a route by which secondary classrooms can prepare students for later probability and statistics while improving near-term reasoning about uncertainty.

Future work should (i) study learning gains for specific targets (e.g., distinguishing $P(A | B)$ vs. $P(B | A)$), (ii) refine scaffolds that support diverse learners inside the ZPD, (iii) expand case libraries beyond diagnostic testing (e.g., reliability, queues, quality control), and (iv) align assessments to reasoning artifacts, not just numeric answers. With these extensions, secondary courses can more reliably cultivate mathematical literacy and lay a realistic runway for students' subsequent study and application of probability.

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